

Magnetization dynamics at elevated temperatures

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Abstract

By using the quantum kinetic approach with the instantaneous local equilibrium approximation, we propose an equation that is capable of addressing magnetization dynamics for a wide range of temperatures. The equation reduces to the Landau-Lifshitz equation at low temperatures and to the paramagnetic Bloch equation at high temperatures. Near the Curie temperature, the magnetization reversal and dynamics depend on both transverse and longitudinal relaxations. We further include the stochastic fields in the dynamic equation in order to take into account fluctuation at high temperatures. Our proposed equation may be broadly used for modeling laser pump-probe experiments and heat assisted magnetic recording.

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I. INTRODUCTION

The phenomenological Landau-Lifshitz (LL) equation is the basis of powerful micromagnetic codes for simulation of magnetic structure and dynamics in magnetic materials. The key component of the LL equation is that magnetization relaxation during dynamic processes is described by a single damping parameter α [1],

$$\frac{d\mathbf{m}}{dt} = -\gamma\mathbf{m} \times \mathbf{H}_{\text{eff}} - \gamma\frac{\alpha}{m}\mathbf{m} \times (\mathbf{m} \times \mathbf{H}_{\text{eff}}) \quad (1)$$

where $\mathbf{m}(\mathbf{r}, t)$ is the magnetization density vector which is a function of space and time, $m = |\mathbf{m}|$ is its magnitude, and \mathbf{H}_{eff} is the effective magnetic field including the magnetic anisotropic, magnetostatic and external fields. The second term on the right side of the equation describes a phenomenological *transverse* relaxation since the magnitude of the magnetization density m is conserved. Such transverse relaxation model is indeed a valid approximation because the magnetization m (the order parameter) of the ferromagnet is nearly independent of the magnetic field as long as the temperature is not too close to the Curie temperature.

Recently, there is an emerging technological need to extend the LL equation to high temperatures in order to model the dynamics near or above the Curie temperature for laser-induced demagnetization (LID) [2, 3] and heat-assisted magnetic recording (HAMR) [4, 5]. Due to the strong fluctuation of the magnetic momentum, one needs to reduce the size of magnetic cells if one continues to use the LL equation to model the magnetization dynamics. When the cell size reduces to the ultimate smallest size of magnetic atoms, the so-called atomistic LL equation which has the same form as the conventional LL equation [6] had been proposed,

$$\frac{d\mathbf{S}_i}{dt} = -\gamma\mathbf{S}_i \times \mathbf{H} - \gamma\alpha\mathbf{S}_i \times (\mathbf{S}_i \times \mathbf{H}) \quad (2)$$

where \mathbf{H} is an effective magnetic field (treated as a c-number) including a random fluctuating field and \mathbf{S}_i is the spin of i th atom which is treated classically. While the above atomistic LL equation might qualitatively capture some of static and dynamic properties near the Curie temperature [7, 8], we point out below that the above atomistic LL equation has several fundamental problems.

First, the spin S_i in ferromagnetic metals such as Ni, Co, Fe and their alloys is usually small. The replacement of the quantum spin by the classical vector severely neglects the

quantum nature of the spin fluctuation of atomic spins. More importantly, the atomistic LL equation, Eq. (2), has no microscopic origin and it is fundamentally incompatible with quantum mechanics. For example, if one takes the case for $S_i = 1/2$ (e.g., Ni). The second ("damping") term of Eq. (2) becomes $\gamma\alpha(\frac{i}{2}\mathbf{S}_i \times \mathbf{H} - \frac{1}{2}\mathbf{H})$ for a quantum spin and thus Eq. (2) becomes

$$\frac{d\mathbf{S}_i}{dt} = -\gamma(1 + i\alpha/2)\mathbf{S}_i \times \mathbf{H} + \gamma\alpha\mathbf{H}/2. \quad (3)$$

This unphysical equation originates from the broken time-reversal symmetry inherited on the atomistic LL equation.

The second difficulty is that the atomistic LL equation is not derivable from an effective Hamiltonian, even at the phenomenological level. If the atomistic LL equation has some validity, a microscopic or an effective Hamiltonian should exist. For example, if we construct a spin Hamiltonian of the form $H' \propto \sum_i \mathbf{S}_i \cdot (\mathbf{H} + \alpha\mathbf{S}_i \times \mathbf{H})$, the equation of motion for \mathbf{S}_i would be $d\mathbf{S}_i/dt = (1/i\hbar)[\mathbf{S}_i, H']$ which results in an additional term compared to Eq. (2) due to non-zero commutation $[\mathbf{S}_i, \alpha\mathbf{S}_i \times \mathbf{H}] \neq 0$. On the other hand, if one takes the phenomenological Hamiltonian as $H' \propto \sum_i \mathbf{S}_i \cdot (\mathbf{H} + \alpha\mathbf{m} \times \mathbf{H})$ where $\mathbf{m} = \langle \mathbf{S}_i \rangle$ (note that $\langle \rangle$ denotes ensemble thermal averaging and thus \mathbf{m} is a c-number), the result dynamics for \mathbf{S}_i would be

$$\frac{d\mathbf{S}_i}{dt} = \frac{1}{i\hbar}[\mathbf{S}_i, H'] = -\gamma\mathbf{S}_i \times \mathbf{H} - \gamma\alpha\mathbf{S}_i \times (\mathbf{m} \times \mathbf{H}). \quad (4)$$

The above equation is precisely the original LL equation after the thermal averaging. Thus, the macroscopic LL equation is derivable from an effective Hamiltonian while the atomistic LL equation is not.

In spite of above conceptual difficulties in the atomistic LL equation, it has been shown that the result derived from the atomistic LL equation with stochastic fields is in agreement with the Monte Carlo simulation [9]. We point out that this agreement is not surprising: for equilibrium properties such as magnetic moment and critical exponents, the calculated results are insensitive to the details of the "damping"; for dynamic properties such as reversal time, the Monte Carlo steps are calibrated to fit the real time in the atomistic stochastic LL equation [10]. Therefore, such agreement should not be interpreted as the proof of the validity of the atomistic LL equation.

In this paper, we propose an effective magnetization dynamic equation for a wide range of temperatures without assuming the presence of the atomistic LL equation for each atomic

spin. By using the equation of motion for the quantum density matrix within the instantaneous local relaxation time approximation [11], we show that the magnetization dynamics for ferromagnets can be cast in the form of the Bloch equation for paramagnetic spins [12]. In Sec.II, we explicitly derive the generalized Bloch equation and show that the equation is consistent with the known dynamics at low and high temperatures. In Sec. III, we analyze the longitudinal and transverse relaxations from our result, and apply our effective equation to study the magnetization reversal processes near Curie temperatures. Finally, we add necessary stochastic fields in the equation to capture the fluctuation of the dynamics.

II. EFFECTIVE DYNAMIC EQUATION FOR FERROMAGNETS

We start with a density operator $\hat{\rho}$ which may be written in the spinor form $\hat{\rho} = \rho_1 + \boldsymbol{\sigma} \cdot \boldsymbol{\rho}_2$ where ρ_1 and $\boldsymbol{\rho}_2$ are spin-independent and spin-dependent density operators, and $\boldsymbol{\sigma}$ is the Pauli matrix vector. Within the instantaneous local relaxation time approximation, the density operator satisfies the quantum kinetic equation [11]

$$\frac{d\hat{\rho}}{dt} = \frac{1}{i\hbar}[\hat{\rho}, \hat{H}] - \frac{\rho_1 - \bar{\rho}_1}{\tau_p} - \boldsymbol{\sigma} \cdot \frac{\boldsymbol{\rho}_2 - \bar{\boldsymbol{\rho}}_2}{\tau_s} \quad (5)$$

where $\bar{\rho}_1$ and $\bar{\boldsymbol{\rho}}_2$ are the *instantaneous local equilibrium* (ILE) densities; they are different from the static equilibrium values. In electron transport theories, these ILE densities depend on the local chemical potential $\mu(\mathbf{r})$ or the local electric field $\mathbf{E}(\mathbf{r}, t)$ and they are in turn related to the densities themselves. For example, for spin dependent electron transport, the inclusion of the spin relaxation (third term of Eq. 5) leads to the well-known spin-diffusion equation for the spin dependent chemical potential (or spin density) [13]. In the present case, these ILE densities are functions of the local effective magnetic field. At a given time, the effective field consists of the ferromagnetic exchange, anisotropy, external, and classical magnetostatic field; we will discuss these fields in more details later. The two relaxation times τ_p and τ_s represent the momentum and spin relaxation times; these two relaxation times control the electron charge diffusion (conductance) and spin diffusion (spin-dependent transport). If we now consider an effective Hamiltonian $\hat{H} = \hat{H}_0 - g\mu\boldsymbol{\sigma} \cdot \mathbf{H}_t(t)$ where μ is the Bohr magneton, \hat{H}_0 is treated as an unperturbed Hamiltonian, we find the self-consistent equation for the magnetization $\mathbf{m} \equiv g\mu\text{Tr}(\boldsymbol{\sigma}\hat{\rho}) = g\mu\text{Tr}\boldsymbol{\rho}_2$ readily from Eq. (5),

$$\frac{d\mathbf{m}}{dt} = -\gamma\mathbf{m} \times \mathbf{H}_t - \frac{\mathbf{m} - \mathbf{m}_{eq}(\mathbf{H}_t)}{\tau_s}. \quad (6)$$

where the ILE magnetization $\mathbf{m}_{eq} = g\mu\bar{\rho}_2$ is identified as the thermal equilibrium value for a given magnetic field \mathbf{H}_t .

At the first sight, Eq. (6) is similar to the well known Bloch equation [12] that has been widely used for understanding nuclear spin resonance experiments. In the Bloch equation, the equilibrium magnetization \mathbf{m}_{eq} is a known equilibrium state which is related to the dynamic susceptibility $\chi(\omega)$, i.e., $\mathbf{m}_{eq} = \chi\mathbf{H}_{ext}$ and \mathbf{m}_{eq} is independent of $\mathbf{m}(t)$. In the present content, \mathbf{m}_{eq} is not known *a priori* and \mathbf{m}_{eq} varies with time. At any time t , there is an instantaneous equilibrium magnetization \mathbf{m}_{eq} that depends on the total magnetic field \mathbf{H}_t . To solve Eq. (6), one first needs to model the instantaneous local field \mathbf{H}_t and its relation to \mathbf{m}_{eq} .

In the conventional LL equation, the effective field \mathbf{H}_{eff} consists of the external field, the anisotropy and the magnetostatic (dipole) fields. The exchange field which comes from the ferromagnetic exchange interaction between neighboring spins is included only when there is spatial variation in the magnetic domain structure. The uniform exchange term, $J\mathbf{m}$, is unimportant since it is parallel to the magnetization and it does not contribute to the LL dynamic equation. In the present case, however, the exchange interaction is the largest and most important term in determining the instantaneous equilibrium magnetization \mathbf{m}_{eq} . We thus model the total instantaneous magnetic field $\mathbf{H}_t = J\mathbf{m} + \mathbf{H}_{eff}$. It is noted that \mathbf{H}_{eff} depends on the instantaneous magnetization $\mathbf{m}(t)$ as well.

Next, we should establish an explicit relation between the total field \mathbf{H}_t with \mathbf{m}_{eq} . There are a number of approaches available to describe such relation. The simplest approach would be using the molecular field approximation where the equilibrium magnetization can be explicitly expressed by [14]

$$\mathbf{m}_{eq} \equiv g\mu \langle \mathbf{S}_i \rangle = g\mu S B_S(\beta g\mu H_t) \hat{\mathbf{H}}_t \quad (7)$$

where S is the spin of the atom, $\beta = (k_B T)^{-1}$ is the inverse of temperature, $B_S(x) \equiv (1/S)[(S + 1/2) \coth((S + 1/2)x) - (1/2) \coth(x/2)]$ is the Brillouin function and $\hat{\mathbf{H}}_t$ is the unit vector in the direction of \mathbf{H}_t , i.e., $\hat{\mathbf{H}}_t = \mathbf{H}_t/H_t$. In the time-independent case, $\mathbf{m} = \mathbf{m}_{eq}$ and the above equation is the well-known mean-field result that determines the ferromagnetic order parameter \mathbf{m}_{eq} . In a non-equilibrium situation where \mathbf{m} depends on time, we interpret \mathbf{m}_{eq} in Eq. (7), which is also dependent on time, as the instantaneous local equilibrium magnetization at a given (instantaneous) field \mathbf{H}_t .

Our proposed Eq. (6) supplemented by Eq. (7) can semi-quantitatively describe magnetization dynamics at all temperatures. Before we examine some limiting cases, we comment on certain important approximations leading to these equations. The instantaneous relaxation time approximation, Eq. (5), has been routinely applied to many quantum or semi-classical systems for transport and magnetic properties. The accuracy of this approximation is hard to assess for the ferromagnetic systems. However, the instantaneous relaxation time approximation has been very successfully applied in spin diffusion of magnetic multilayers where the semiclassical distribution function is assumed to relax to the instantaneous chemical potential [13]. Furthermore, the relaxation time approximation usually serves as a first step in a phenomenological theory since it gives rise to an analytically closed form. The most severe approximation is to replace \mathbf{m}_{eq} by the mean field Brillouin function, Eq. (7). Such approximations are known to produce inaccurate critical exponents and Curie temperatures. There are several much improved approaches such as the renormalization group theory [15], self-consistent random phase approximation [16], and Monte Carlo simulation [17]. While these approaches treat the fluctuation near the critical temperature better, they are far more complicated and without an analytical form. On the other hand, the mean field approximation is qualitatively correct and it allows a much simpler description of magnetization dynamics in spite of underestimating the critical fluctuation. For the purpose of establishing a phenomenological dynamic equation similar to the LL equation, we believe that the choice of the mean field approximation throughout this study is appropriate.

Similar to the LL equation, Eq. (6) contains a phenomenological parameter, τ_s , representing the magnetic relaxation of paramagnetic spins. In transition metals, τ_s is related to the spin-flip time. In fact, there are a number of theoretical and experimental studies on the numerical values of τ_s in different materials [18–20]. For transition metals, the relaxation time ranges from sub-picoseconds to a few picoseconds.

III. LONGITUDINAL AND TRANSVERSE MAGNETIZATION DYNAMICS

Before we proceed to solve Eq. (6) in a number of interesting examples, we examine several limiting cases. First, by using the identity

$$\mathbf{H}_t = m^{-2}[(\mathbf{m} \cdot \mathbf{H}_t)\mathbf{m} - \mathbf{m} \times (\mathbf{m} \times \mathbf{H}_t)] \quad (8)$$

we write Eq. (6) in terms of three mutually perpendicular vectors,

$$\frac{d\mathbf{m}}{dt} = -\gamma\mathbf{m} \times \mathbf{H}_{\text{eff}} - \frac{\gamma\alpha_{\text{tr}}}{m}\mathbf{m} \times (\mathbf{m} \times \mathbf{H}_{\text{eff}}) - \frac{\gamma\alpha_l}{m}(\mathbf{m} \cdot \mathbf{H}_t)\mathbf{m} \quad (9)$$

where we have introduced the transverse and longitudinal dimensionless damping coefficients α_{tr} and α_l ,

$$\alpha_{\text{tr}} = \frac{m_{\text{eq}}}{\gamma\tau_s m H_t} \quad (10)$$

and

$$\alpha_l = \frac{1}{\gamma\tau_s} \left[\frac{m}{\mathbf{m} \cdot \mathbf{H}_t} - \frac{m_{\text{eq}}}{m H_t} \right]. \quad (11)$$

At low temperatures, \mathbf{m} is close to $g\mu S$ and the exchange field $J\mathbf{m}$ is much larger than the other fields \mathbf{H}_{eff} . Thus, one immediately has $\alpha_{\text{tr}} = (\gamma\tau_s J m)^{-1}$. In a typical transition ferromagnet such as Co or Fe, J is of the order of the Curie temperature (0.1-0.2 eV) and τ_s is a sub-picosecond, we find α_{tr} is of the order of $10^{-3} - 10^{-1}$.

To estimate the low temperature longitudinal relaxation α_l from Eq. (11), we consider an initial m deviates from the equilibrium value of $g\mu S B_S$ and from Eq. (11), α_l would be about the same order of magnitude as α_{tr} . However, the longitudinal field $J\mathbf{m}$ is much larger than H_{eff} and thus the ratio of the longitudinal (τ_l) to the transverse (τ_{tr}) relaxation times is about $\tau_l/\tau_{\text{tr}} \approx H_{\text{eff}}/J$. Even for a very high anisotropy material and a large magnetic field, J is several orders of magnitude larger than H_{eff} ; this justifies that at the low temperature one can neglect the longitudinal relaxation in the dynamic equation, i.e., the magnitude of the magnetization is always in equilibrium.

When the temperature is much higher than the Curie temperature, Eq. (6) represents the paramagnetic Bloch equation. In this case, the equilibrium magnetization \mathbf{m}_{eq} may be expressed via susceptibility χ , i.e., $\mathbf{m}_{\text{eq}} = \chi\mathbf{H}_{\text{eff}}$. Such dynamic equations have been frequently used for understanding paramagnetic resonant phenomena where the resonance width is determined by the relaxation time τ_s .

The most interesting case of Eq. (6) is for temperature close to Curie temperature where transverse and longitudinal relaxation times could become comparable. To see this, we consider the effective field is parallel to $\mathbf{m}(t) = m(t)\mathbf{e}_z$ and expand $B_S(x) = (S+1)x/3 - (1/90)(S+1)(2S^2+2S+1)x^3$ up to the third order in the small x where $x = \beta g\mu H_t$. Then, Eq. (6) for temperature close to the Curie temperature becomes

$$\frac{dm}{dt} = -\frac{1}{J\tau_s} \left[\left(1 - \frac{T_c}{T}\right) H_t + \frac{3}{10J^2} \frac{T_c^3}{T^3} \left(\frac{1}{S^2} + \frac{1}{(1+S)^2} \right) H_t^3 - H_{\text{eff}} \right] \quad (12)$$

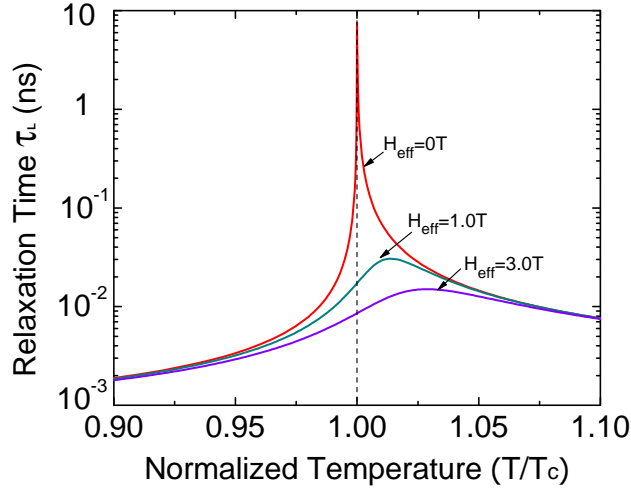


FIG. 1: (Color online) The longitudinal relaxation time τ_l as a function of temperature for several magnetic fields. We choose a small difference between m and m_{eq} at $t = 0$ and identify $t = \tau_l$ where the difference is reduced by the half. We have used $\tau_s = 1$ ps and $S = 1/2$.

where $T_c = S(S+1)J(g\mu)^2/3k_B$ is the mean field Curie temperature. In the absence of the magnetic field $\mathbf{H}_{\text{eff}} = 0$ and $H_t = Jm$, and we can immediately solve the above equation,

$$m(t) = m(0)e^{-t/\tau_l} [1 + G(1 - e^{-2t/\tau_l})]^{-1/2} \quad (13)$$

where $m(0)$ is the initial magnetization, $G = \frac{3T_c^3 m^2(0)}{10T^3} \left(\frac{1}{S^2} + \frac{1}{(1+S)^2} \right) (1 - T_c/T)^{-1}$, and

$$\tau_l = \tau_s \left(1 - \frac{T_c}{T} \right)^{-1}. \quad (14)$$

Thus, the longitudinal relaxation time, $|\tau_l|$, near Curie temperature, is associated with the critical phenomenon. The relaxation time becomes very long when the temperature approaches the Curie temperature. The dynamics slow-down at the critical temperature is in fact a general property of critical phenomena [21]. In the presence of the magnetic field, the phase transition becomes a smooth change and the dynamic slow-down is no more critical. In Fig. 1, we show the longitudinal relaxation as the function of the magnetic field and temperature. Clearly, the magnetic field suppresses the longitudinal dynamic slowdown. It is noted that the peak of the relaxation time in the presence of the magnetic field is shifted to higher temperatures.

In order to gain more quantitative insight for the interplay between the transverse and longitudinal relaxations, we consider several simple cases where the numerical calculations

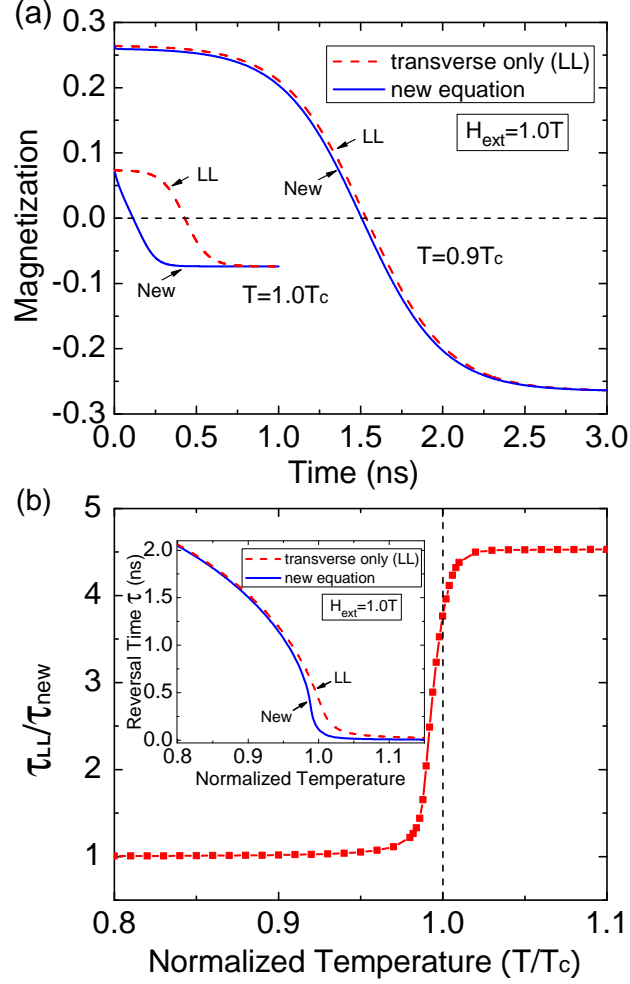


FIG. 2: (Color online) a) The time dependence of magnetization reversal when a reversal magnetic field is applied at $t > 0$ for the temperature $T = 0.9T_c$ and $T = 1.0T_c$ as indicated. The blue (solid) and red (dashed) curves were obtained by Eq. (6) and by the first two terms of Eq. (9), respectively. b) the reversal times and their ratio as a function of the temperature obtained from a). The parameters are $\tau_s = 1.0$ ps, $S = 1/2$, the anisotropy constant $K = 0$, and the external field $H = 1.0$ T.

can be readily performed. We assume that the magnetic particle is a single domain so that there is no spatial dependence of the effective field and the magnetization. Furthermore, the long-range magnetostatic field is also discarded. In the first case, we compare the reversal times with and without the longitudinal relaxation in a simplest case: an isotropic magnetic particle (zero magnetic anisotropy) is initially magnetized at 5° from $+z$ axis and a reversal magnetic field in the direction of $-z$ is applied at $t > 0$. Figure 2(a) shows the importance

of the longitudinal relaxation when the temperature approaches Curie temperature. We compare the magnetization dynamics with and without the last term of Eq. (9). If the temperature is considerably below the Curie temperature, e.g., $T = 0.9T_c$, the longitudinal relaxation term has a negligible effect, i.e., the result is essentially same whether the last term of Eq. (9) is included. This is because the magnitude of the magnetization is nearly time-independent at low temperature. When the temperature is near the Curie temperature, the magnitude of the magnetization is significantly reduced. More importantly, the magnitude is now a function of time due to its dependence on the total effective field. In this case, there is a much difference if one includes the longitudinal relaxation. In Fig. 2(b), we show the ratio of the reversal times calculated with and without the longitudinal relaxation. Clearly, the reversal time from Eq. (6) is much faster than that of the LL equation if the temperature is close to or higher than Curie temperature.

Next we apply our equation to a hypothetical HAMR process when the laser heating and thermal diffusion produce a time-dependent temperature profile: the temperature of the particle increases linearly $T(t) = T_{\text{rm}} + (t/t_{\text{heat}})(T_{\text{p}} - T_{\text{rm}})$ from the room temperature T_{rm} to a peak value T_{p} for the period of $0 < t < t_{\text{heat}}$ of lasing application. After the heating process is completed and the laser is removed, the temperature decreases due to heat diffusion into surroundings. We assume the temperature is $T(t) = T_{\text{rm}} + (T_{\text{p}} - T_{\text{rm}}) \exp[-(t - t_{\text{heat}})/t_{\text{cool}}]$ for $t > t_{\text{heat}}$. While the precise temperature profile should be determined via heat transport equations with proper boundary conditions, our hypothetical temperature is characterized by three parameters: the peak temperature of the particle T_{p} , and the heating and cooling rates $1/t_{\text{heat}}$ and $1/t_{\text{cool}}$. We choose the low-temperature magnetic anisotropy field much larger than the external magnetic field so that the magnetic reversal does not occur at the room temperatures. The temperature dependence of the anisotropy energy E_a is modeled by $E_a = Km^2(T) \sin^2 \theta$, where $m(T)$ is the magnitude of the magnetization at temperature T and θ is the angle between the magnetization vector and z -axis [22, 23]. By placing the above temperature profile and effective magnetic fields into Eq. (6), we have numerically calculated the time dependent magnetization shown in Fig. (3). As we expected, the magnetization reversal requires a high peak temperature T_{p} to reduce the anisotropy. The rates of heating and cooling are also important; they should be slow enough so that the magnetization has sufficient time to relax to the ground state via transverse and longitudinal relaxations.

More quantitatively, we have made two comparisons in Fig. 3. First, we compare our

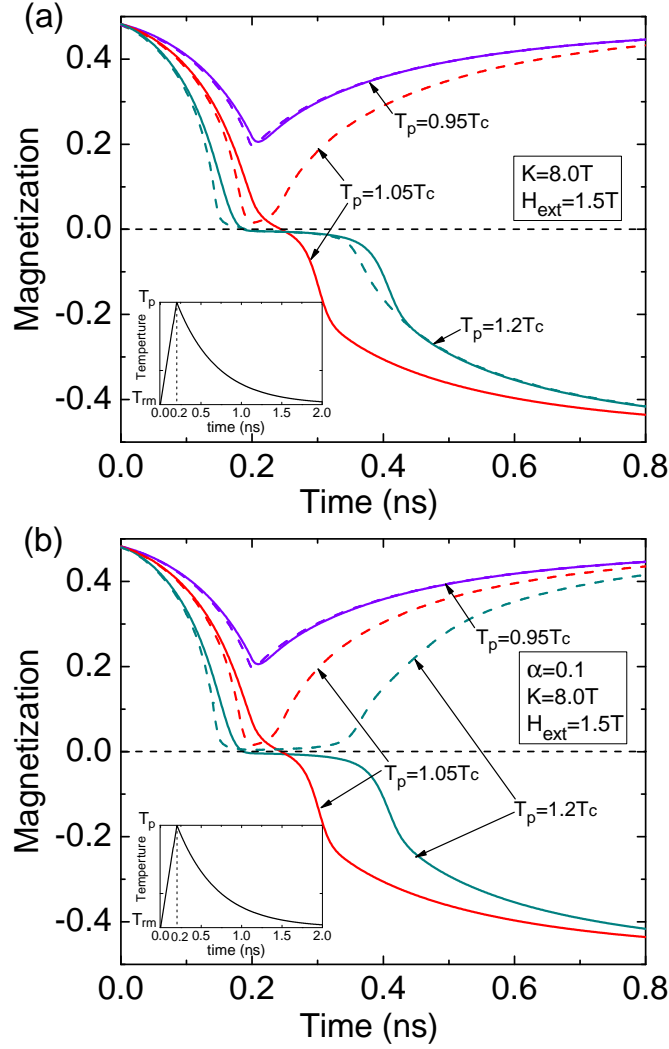


FIG. 3: (Color online) Time dependence of magnetization for given temperature profiles after a reversal magnetic field $H = 1.5T$ is applied for $t > 0$. a) The results are obtained from Eq. (6) (solid curves) and from Eq. (15) (dashed curves). b) Comparison of the results from Eq. (6) and from the conventional LL with a constant damping parameter (dashed curves). The Inserts are the hypothetical temperature profiles. The parameters are $K = 8.0T$, $S = 1/2$, and $\tau_s = 1.0$ ps.

equation with a modified LL which allows the magnitude of the magnetization varying with the time due to changing temperature in HAMR,

$$\frac{d\mathbf{m}}{dt} = \frac{d(m_{eq}\hat{\mathbf{m}})}{dt} = -\gamma\mathbf{m} \times \mathbf{H}_{eff} - \gamma\frac{\alpha_{tr}}{m}\mathbf{m} \times (\mathbf{m} \times \mathbf{H}_{eff}) + \frac{dm_{eq}}{dT} \cdot \frac{dT}{dt}\hat{\mathbf{m}} \quad (15)$$

where the transverse damping parameter is given by Eq. (10). The above equation implies that the magnitude of the magnetization is always in equilibrium with the instantaneous

temperature, i.e., the longitudinal relaxation is infinite fast. Fig. 3(a) shows that such approximation is quite accurate even for the temperature $T_p = 0.95T_c$. However, the deviation begins to show up when the peak temperature is higher than the Curie temperature. In Fig. 3(b), we further compare our results with a constant damping parameter (i.e., taking α_{tr} in Eq. (15) as a constant). The deviation of this conventional LL with ours becomes more significant at high temperatures. For example, even for $T_p = 1.2T_c$, the magnetization reversal is not possible from the conventional LL equation, see Fig. 3(b).

To end this section, we should briefly compare our equation with the LLB equation of Garanin [6]. Since the LLB equation is based on the atomistic LL equation that we believe is questionable, we should not make extensive comparisons. We point out that the LLB equation also contains the transverse and longitudinal relaxations, and the essential difference is the temperature dependence of the relaxation parameters. At low temperatures, both our equation and the LLB equation reduce to the conventional LL equation. At high temperatures, the relaxations in the LLB equation depend explicitly on temperatures; this is because the longitudinal relaxation to the equilibrium magnetization is solely controlled by the classical random field which is proportional to the temperature. In our case, the dependence of the relaxation time on temperature is implicit, via the temperature dependence of the equilibrium magnetization. More importantly, the instantaneous relaxation time τ_s in our equation has microscopical meaning as the scattering lifetime of electron spins while the damping parameter in the atomistic LL does not have a microscopic counterpart.

IV. STOCHASTIC FIELDS

Our proposed equation, Eq. (6), describes the time-dependence of the average magnetization. The fluctuation at the finite temperature, particularly at a high temperature, becomes important. To address the fluctuation, one should include stochastic fields in the macroscopic dynamic equation. Similar to Brown's method [24] for the LL equation, we introduce the stochastic fields $\mathbf{h}(t)$ as follows,

$$\frac{d\mathbf{m}}{dt} = -\gamma\mathbf{m} \times (\mathbf{H}_{\text{eff}} + \mathbf{h}) - \frac{\mathbf{m} - \mathbf{m}_{\text{eq}}}{\tau_s}. \quad (16)$$

We point out that the stochastic field does not enter in the relaxation term although the instantaneous equilibrium magnetization depends on the total field \mathbf{H}_t . The reason is as

follows. The interaction between the random field and the magnetization is $-\mathbf{m} \cdot \mathbf{h}(t)$. This interaction gives rise to a random torque on the magnetization $-\gamma \mathbf{m} \times \mathbf{h}(t)$ that is added to the deterministic torque equation. As in the case of the Brownian motion, the Langevin random field $\mathbf{f}(t)$ is only included in the particle motion $d^2r/dt^2 = -\alpha \mathbf{v} + \mathbf{F}(t) + \mathbf{f}(t)$ (where the friction force $-\alpha \mathbf{v}$ and the external driven force $\mathbf{F}(t)$ are not changed by the random force). To determine the magnitude and the correlation of the stochastic fields, we first write the above stochastic equation in the standard form of Langevin,

$$\frac{dm_i}{dt} = \left[-\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} - \frac{\mathbf{m} - \mathbf{m}_{\text{eq}}}{\tau_s} \right]_i - \gamma \sum_{jk} \varepsilon_{ijk} m_j h_k. \quad (17)$$

where ε_{ijk} is the Levi-Civita symbol. The corresponding Fokker-Planck equation is thus have the following form,

$$\frac{\partial P}{\partial t} = - \sum_i \frac{\partial}{\partial m_i} \left[\left(-\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} - \frac{\mathbf{m} - \mathbf{m}_{\text{eq}}}{\tau_s} \right)_i + D \gamma^2 \mathbf{m} \times \left(\mathbf{m} \times \frac{\partial}{\partial \mathbf{m}} \right)_i \right] P \quad (18)$$

where P is the probability density and D is the random field correlation constant. At the equilibrium, one may assume that the probability density takes a simple Boltzmann distribution, i.e., $P \propto \exp(-\mathbf{m} \cdot \mathbf{H})$. By placing this form of P into Eq. (18), one finds the desired correlation of the random field given below,

$$\langle h_i(t) h_j(0) \rangle = \frac{2k_B T \alpha_{\text{tr}}}{\gamma m V} \delta_{ij} \delta(t) \quad (19)$$

V. SUMMARY

In this paper, we have proposed a model of magnetization dynamics for an entire range of temperature based on the quantum kinetic approach with the instantaneous local relaxation time approximation. The resulting equation generates a low temperature magnetization dynamic same as the Landau-Lifshitz equation, namely, the transverse magnetization is sufficient to describe dynamics. When the temperature approaches or exceeds the Curie temperature, it is essential to include the longitudinal magnetization relaxation. With our new dynamic equation, one can model the entire heat-assisted magnetic recording processes when the temperature are heated and cooled through the Curie temperature [25–27]. The stochastic fields on the magnetization are also proposed. This work is partially supported

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